

Class XI Session 2025-26

Subject - Mathematics

Sample Question Paper - 2

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If $\sec \theta = \sqrt{2}$ and $\frac{3\pi}{2} < \theta < 2\pi$, then $\frac{(1+\tan \theta + \operatorname{cosec} \theta)}{(1+\cot \theta - \operatorname{cosec} \theta)} = ?$ [1]
a) -1
b) $\frac{3}{4}$
c) $\frac{-3}{8}$
d) $\frac{\sqrt{3}}{2}$
2. Let A and B be finite sets containing m and n elements respectively. The number of relations that can be defined from A to B is [1]
a) 2^{m+n}
b) 0
c) mn
d) 2^{mn}
3. The mean of five numbers is 30. If one number is excluded, their mean becomes 28. The excluded number is: [1]
a) 35
b) 38
c) 30
d) 28
4. $\lim_{\theta \rightarrow \pi/2} \frac{1-\sin \theta}{(\pi/2-\theta) \cos \theta}$ is equal to [1]
a) 1
b) $\frac{1}{2}$
c) -1
d) $-\frac{1}{2}$
5. A line cutting off intercept - 3 from the y-axis and the tangent at angle to the x-axis is $\frac{3}{5}$, its equation is [1]



- a) $5y + 3x - 15 = 0$ b) $5y - 3x - 15 = 0$
c) $5y - 3x + 15 = 0$ d) $3y - 5x + 15 = 0$
6. The distance of point P(a, b, c) from y-axis is: [1]
a) b b) b^2
c) $a^2 + c^2$ d) $\sqrt{a^2 + c^2}$
7. If $\frac{1-ix}{1+ix} = a + ib$, then $a^2 + b^2 =$ [1]
a) -1 b) 2
c) 0 d) 1
8. How many different committees of 5 can be formed from 6 men and 4 women on which exact 3 men and 2 women serve? [1]
a) 20 b) 120
c) 6 d) 60
9. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x}$ is equal to: [1]
a) $\frac{1}{2}$ b) 1
c) 2 d) 0
10. $(2 \cos^2 15^\circ - 1) = ?$ [1]
a) $\frac{3}{2}$ b) $\frac{\sqrt{3}}{\sqrt{2}}$
c) $\frac{\sqrt{3}}{2}$ d) $2\sqrt{3}$
11. Which of the following is a null set? [1]
a) $A = \{x : x > 1 \text{ and } x < 3\}$ b) $B = \{x : x + 3 = 3\}$
c) $D = \{0\}$ d) $C = \phi$
12. The number of terms in the expansion of $\left(3x - \frac{5}{y}\right)^{10}$ is [1]
a) 9 b) 11
c) 8 d) 10
13. The coefficient of x in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is [1]
a) $12a^2$ b) $9a^4$
c) $7a^4$ d) $10a^3$
14. Solve the system of inequalities $x - 2 > 0, 3x < 18$ [1]
a) $-6 < x < -2$ b) $2 < x < 6$
c) $1 < x < 3$ d) $3 < x < 18$
15. The smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is [1]
a) $\{1, 2, 5, 9\}$ b) $\{2, 3, 5\}$
c) $\{4, 5, 6\}$ d) $\{3, 5, 9\}$

16. $\cos 50^\circ \cos 10^\circ - \sin 50^\circ \sin 10^\circ = ?$ [1]
 a) 1 b) $\frac{1}{\sqrt{2}}$
 c) $\frac{\sqrt{3}}{2}$ d) $\frac{1}{2}$
17. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$ is equal to [1]
 a) 0 b) $\frac{1}{2}$
 c) 1 d) 2
18. The number of arrangements that can be formed by all the letters of the word LAUGHTER is [1]
 a) 5040 b) 32768
 c) 20160 d) 40320
19. **Assertion (A):** Let $A = \{a, b\}$ and $B = \{a, b, c\}$. Then, $A \not\subset B$. [1]
Reason (R): If $A \subset B$, then $A \cup B = B$.
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
 c) A is true but R is false. d) A is false but R is true.
20. **Assertion (A):** If the numbers $\frac{-2}{7}$, K , $\frac{-7}{2}$ are in GP, then $k = \pm 1$. [1]
Reason (R): If a_1, a_2, a_3 are in GP, then $\frac{a_2}{a_1} = \frac{a_3}{a_2}$.
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
 c) A is true but R is false. d) A is false but R is true.

Section B

21. Let $f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = x^2$. Determine: $\{x : f(x) = 4\}$ [2]
 OR
 Find the domain of the function f defined by $f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$.
22. If $y = \sin(\sqrt{\sin x + \cos x})$ find $\frac{dy}{dx}$. [2]
23. A die is thrown. Find the probability of getting a number less than or equal to 4. [2]

OR

An experiment involves tossing of two coins and recording them in the following events

A: no tail

B: exactly one fail

C: at least one tail

write the sets representing events (i) A and C (ii) A but not B

24. If $A = \{a, b, c, d, e\}$, $B = \{a, c, e, g\}$ and $C = \{b, e, f, g\}$, verify that: $B \cap C = C \cup B$ [2]
25. Find the equation of a line which makes an angle of $\tan^{-1}(3)$ with the x-axis and cuts off an intercept of 4 units on negative direction of y-axis. [2]

Section C

26. Find the number of different words that can be formed from the letters of the word TRIANGLE, so that no vowels are together. [3]
27. Show that the points $(-2, 3, 5)$, $(1, 2, 3)$ and $(7, 0, -1)$ are collinear. [3]

28. show that the coefficient of the middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is 252. [3]

OR

Find the coefficient of x^6 in the expansion of $\left(3x^2 - \frac{1}{3x}\right)^9$.

29. Evaluate: $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2+1}-1}$. [3]

OR

Find the value of **a** and **b** if $\lim_{x \rightarrow 2} f(x)$ and $\lim_{x \rightarrow 4} f(x)$ exists where $f(x) = \begin{cases} x^2 + ax + b & , 0 \leq x < 2 \\ 3x + 2 & , 2 \leq x \leq 4 \\ 2ax + 5b & , 4 < x \leq 8 \end{cases}$

30. The product of three numbers in G.P. is 125 and the sum of their products taken in pairs is $87\frac{1}{2}$. Find them. [3]

OR

Find the 10th and nth terms of the GP $\frac{-3}{4}, \frac{1}{2}, \frac{-1}{3}, \frac{2}{9}, \dots$

31. Let $A = \{2, 4, 6, 8, 10\}$, $B = \{4, 8, 12, 16\}$ and $C = \{6, 12, 18, 24\}$. Using Venn diagrams, verify that: $(A \cup B) \cup C = A \cup (B \cup C)$ [3]

Section D

32. Calculate mean, variance and standard deviation of the following frequency distribution: [5]

Class:	0-10	10-20	20-30	30-40	40-50	50-60
Frequency:	11	29	18	4	5	3

33. Find the lengths major and minor axes, coordinates of the vertices, coordinates of the foci, eccentricity, and length of the latus rectum of the ellipse. $3x^2 + 2y^2 = 18$. [5]

OR

The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100 m long is supported by vertical wires attached to the cable, the longest wire being 30 m and the shortest being 6 m. Find the length of a supporting wire attached to the roadway 18 m from the middle.

34. Solve the following system of linear inequalities [5]

$$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4} \text{ and } \frac{7x-1}{3} - \frac{7x+2}{6} > x.$$

35. Prove that $\cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15} = \frac{1}{16}$ [5]

OR

If $\cos x = -\frac{3}{5}$ and x lies in the IIIrd quadrant, find the values of $\cos \frac{x}{2}$, $\sin \frac{x}{2}$ and $\sin 2x$.

Section E

36. Read the following text carefully and answer the questions that follow: [4]

Method to Find the Sets When Cartesian Product is Given

For finding these two sets, we write first element of each ordered pair in first set say A and corresponding second element in second set B (say).

Number of Elements in Cartesian Product of Two Sets

If there are p elements in set A and q elements in set B, then there will be pq elements in $A \times B$ i.e. if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

- The Cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$. (1)
- A and B are two sets given in such a way that $A \times B$ contains 6 elements. If three elements of $A \times B$ are $(1, 3)$, $(2, 5)$ and $(3, 3)$, then find the remaining elements of $A \times B$. (1)



iii. If the set A has 3 elements and set B has 4 elements, then find the number of elements in $A \times B$. (2)

OR

If $A \times B = \{(a, 1), (b, 3), (a, 3), (b, 1), (a, 2), (b, 2)\}$. Find A and B. (2)

37. **Read the following text carefully and answer the questions that follow:**

[4]

Two students Ankit and Vinod appeared in an examination. The probability that Ankit will qualify the examination is 0.05 and that Vinod will qualify is 0.10. The probability that both will qualify is 0.02.

- Find the probability that atleast one of them will qualify the exam. (1)
- Find the probability that atleast one of them will not qualify the exam. (1)
- Find the probability that both Ankit and Vinod will not qualify the exam. (2)

OR

Find the probability that only one of them will qualify the exam. (2)

38. **Read the following text carefully and answer the questions that follow:**

[4]

The conjugate of a complex number z , is the complex number, obtained by changing the sign of imaginary part of z . It is denoted by \bar{z} .

The modulus (or absolute value) of a complex number, $z = a + ib$ is defined as the non-negative real number $\sqrt{a^2 + b^2}$. It is denoted by $|z|$. i.e.

$$|z| = \sqrt{a^2 + b^2}$$

Multiplicative inverse of z is $\frac{\bar{z}}{|z|^2}$. It is also called reciprocal of z .

$$z\bar{z} = |z|^2$$

- If $f(z) = \frac{7-z}{1-z^2}$, where $z = 1 + 2i$, then find $|f(z)|$. (1)
- Find the value of $(z + 3)(\bar{z} + 3)$. (1)
- If $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$, then find the value of $x + y$. (2)

OR

If $z = 3 + 4i$, then find \bar{z} . (2)



Solution

Section A

1. (a) -1

Explanation:

In quadrant IV, $\sin \theta < 0$.

$$\sec \theta = \sqrt{2} \Rightarrow \cos \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \sin^2 \theta = (1 - \cos^2 \theta) = (1 - \frac{1}{2}) = \frac{1}{2} \Rightarrow \sin \theta = \frac{-1}{\sqrt{2}}$$

$$\therefore \tan \theta = \frac{\sin \theta}{\cos \theta} = -1, \cot \theta = -1, \operatorname{cosec} \theta = -\sqrt{2}$$

$$\therefore \frac{(1 + \tan \theta + \operatorname{cosec} \theta)}{(1 + \cot \theta - \operatorname{cosec} \theta)} = \frac{(1 - 1 - \sqrt{2})}{(1 - 1 + \sqrt{2})} = \frac{-\sqrt{2}}{\sqrt{2}} = -1.$$

2.

(d) 2^{mn}

Explanation:

We have $n(A) = m, n(B) = n$.

\therefore Number of relations defined from A to B

$$= \text{number of possible subsets of } A \times B = 2^{n(A \times B)} = 2^{mn}$$

3.

(b) 38

Explanation:

Let the numbers are x_1, x_2, x_3, x_4 and x_5 . Then,

$$\text{we have, } \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 30$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 150 \dots(i)$$

$$\text{Now, suppose } x_1 \text{ is excluded, then } \frac{x_2 + x_3 + x_4 + x_5}{4} = 28 \text{ [given]}$$

$$\Rightarrow x_2 + x_3 + x_4 + x_5 = 112 \dots(ii)$$

From Eqs. (i) and (ii), we get $x_1 = 150 - 112 = 38$

4.

(b) $\frac{1}{2}$

Explanation:

$$\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 - \sin \theta}{\left(\frac{\pi}{2} - \theta\right) \cos \theta}$$

$$= \lim_{h \rightarrow 0} \frac{1 - \cos h}{\left(\frac{\pi}{2} - \left(\frac{\pi}{2} - h\right)\right) \sin h}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2}}{h \sin h}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2}}{\frac{4h^2}{\frac{\sin h}{h}}}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

5.

(c) $5y - 3x + 15 = 0$

Explanation:



Here, it is given that

$$\tan \theta = \frac{3}{5}$$

We know that,

Slope of a line, $m = \tan \theta$

$$\Rightarrow \text{Slope of line, } m = \frac{3}{5}$$

Since, the lines cut off intercepts -3 on y -axis then the line is passing through the point $(0, -3)$.

Therefore, the equation of line is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - (-3) = \frac{3}{5}(x - 0)$$

$$\Rightarrow y + 3 = \frac{3}{5}x$$

$$\Rightarrow 5y + 15 = 3x$$

$$\Rightarrow 5y - 3x + 15 = 0$$

6.

(d) $\sqrt{a^2 + c^2}$

Explanation:

$$\sqrt{a^2 + c^2}$$

7.

(d) 1

Explanation:

$$1$$

$$\frac{1-ix}{1+ix} = a + ib$$

Taking modulus on both the sides, we get:

$$\left| \frac{1-ix}{1+ix} \right| = |a + ib|$$

$$\Rightarrow \frac{\sqrt{1^2+x^2}}{\sqrt{1^2+x^2}} = \sqrt{a^2+b^2}$$

$$\Rightarrow \sqrt{a^2+b^2} = 1$$

Squaring both the sides, we get:

$$a^2 + b^2 = 1$$

8.

(b) 120

Explanation:

Number of commotes that can be formed $= {}^6C_3 \times {}^4C_2$

$$= \frac{6!}{3!3!} \times \frac{4!}{2!2!}$$

$$= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{4 \times 3}{2}$$

$$= 120.$$

9. (a) $\frac{1}{2}$

Explanation:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{(\sqrt{1+x}+1)x}$$

$$= \lim_{x \rightarrow 0} \frac{1+x-1}{x\sqrt{1+x}+1}$$

$$= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{1+x}+1)}$$

$$= \frac{1}{2}$$

10.

(c) $\frac{\sqrt{3}}{2}$



Explanation:

Using $(2 \cos^2 \theta - 1) = \cos 2\theta$, we get

$$(2 \cos^2 15^\circ = \cos(2 \times 15^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2})$$

11.

(d) $C = \phi$

Explanation:

ϕ is denoted as null set.

12.

(b) 11

Explanation:

Therefore, the expansion of $\left(3x - \frac{5}{y}\right)^{10}$ has 11 terms.

For $(x+a)^n$, number of terms = $n+1$

$n=10$ hence terms is $10+1=11$.

13.

(d) $10a^3$

Explanation:

In the expansion of $\left(x^2 + \frac{a}{x}\right)^5$, we have

$$T_{r+1} = {}^5C_r \cdot (x^2)^{(5-r)} \cdot \left(\frac{a}{x}\right)^r = {}^5C_r \cdot x^{(10-3r)} \cdot a^r.$$

Substituting $10 - 3r = 1$, we get $3r = 9 \Rightarrow r = 3$.

$$\therefore \text{coefficient of } x = {}^5C_r \cdot a^3 = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} a^3 = 10a^3.$$

14.

(b) $2 < x < 6$

Explanation:

$$x - 2 > 0$$

$$\Rightarrow x > 2$$

$$\Rightarrow x \in (2, \infty)$$

$$\text{Now } 3x < 18$$

$$\Rightarrow x < 6$$

$$\Rightarrow x \in (-\infty, 6)$$

$$\text{So solution set is } (2, \infty) \cap (-\infty, 6) = (2, 6)$$

$$\Rightarrow 2 < x < 6$$

15.

(d) $\{3, 5, 9\}$

Explanation:

The union of two sets A and B is the set of elements in A, or B, or both.

So smallest set A = $\{3, 5, 9\}$

16.

(d) $\frac{1}{2}$

Explanation:

$$\cos 50^\circ \cos 10^\circ - \sin 50^\circ \sin 10^\circ = \cos (50^\circ + 10^\circ) [\because \cos A \cos B - \sin A \sin B = \cos (A + B)]$$

$$= \cos 60^\circ = \frac{1}{2}$$

17.

(d) 2

Explanation:

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sin 2x}{x} \\&= \lim_{x \rightarrow 0} 2 \left(\frac{\sin 2x}{2x} \right) \\&= 2 \times 1 \\&= 2\end{aligned}$$

18.

(d) 40320

Explanation:

The word LAUGHTER consists of 8 letters which are all distinct.

Hence the required number of arrangements = ${}^8P_8 = 8! = 40320$

19.

(d) A is false but R is true.

Explanation:

Assertion $A = \{a, b\}$, $B = \{a, b, c\}$

Since, all the elements of A are in B. So,

$A \subset B$

Reason $\because A \subset B$

$\Rightarrow A \cup B = B$

Hence, Assertion is false and Reason is true.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion: If $\frac{-2}{7}$, K, $\frac{-7}{2}$ are in G.P.

Then, $\frac{a_2}{a_1} = \frac{a_3}{a_2}$

[\because common ratio (r) = $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \dots$]

$$\therefore \frac{\frac{-2}{7}}{\frac{-2}{7}} = \frac{\frac{-7}{2}}{k}$$

$$\Rightarrow \frac{7}{-2} k = \frac{-7}{2} \times \frac{1}{k}$$

$$\Rightarrow 7k \times 2k = -7 \times (-2)$$

$$\Rightarrow 14k^2 = 14$$

$$\Rightarrow k^2 = 1 \Rightarrow k = \pm 1$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

Section B

21. Here $f(x) = x^2$

Now, Given, $f(x) = 4$

Thus, $x^2 = 4$

$\Rightarrow x = \pm 2$

Thus, $\{x : f(x) = 4\} = \{-2, 2\}$

OR

As we have, $f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$

For domain of $f(x)$

$4-x \geq 0$ and $x^2-1 > 0$

$\Rightarrow x \leq 4$ and $x^2 > 1$

$\Rightarrow x \leq 4$ and $x \in (-\infty, -1) \cup (1, \infty)$

$\therefore x \in (-\infty, -1) \cup (1, 4]$

22. First of all ,

Put $(\sin x + \cos x) = t$ and $\sqrt{(\sin x + \cos x)} = \sqrt{t} = u$ we get. $y = \sin u$, $u = \sqrt{t}$ and $t = \sin x + \cos x$

$$\frac{dy}{du} = \cos u, \frac{du}{dt} = \frac{1}{2} t^{-1/2} = \frac{1}{2\sqrt{t}}$$

$$\text{And } \frac{dt}{dx} = (\cos x - \sin x)$$

$$\text{so, } \frac{dy}{dx} = \left(\frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx} \right) = \frac{\cos u}{2\sqrt{t}} \cdot (\cos x - \sin x)$$

$$= \frac{\cos \sqrt{t}}{2\sqrt{t}} \cdot (\cos x - \sin x)$$

$$= \frac{\cos(\sqrt{\sin x + \cos x})(\cos x - \sin x)}{2\sqrt{\sin x + \cos x}}.$$

23. We have to find the probability of getting a number less than or equal to 4.

A number less than or equal to 4 is obtained, if we get any one of 1, 2, 3, 4 as an outcome.

So, favourable number of elementary events = 4

$$\text{Hence, required probability} = \frac{4}{6} = \frac{2}{3}.$$

OR

When we toss two coins, the sample space is-

$$S = \{HH, HT, TH, TT\}.$$

$$A = \{H H\}, B = \{HT, TH\},$$

$$C = \{HT, TH, TT\}$$

$$\text{i. } A \text{ and } C = A \cap C = \phi$$

$$\text{ii. } A \text{ but not } B = A - B = \{HH\}$$

24. Suppose x be an element of $B \cap C$

$$\Rightarrow x \in B \cap C$$

$$\Rightarrow x \in B \text{ and } x \in C$$

$$\Rightarrow x \in C \text{ and } x \in B \text{ [by definition of intersection]}$$

$$\Rightarrow x \in C \cap B$$

$$\Rightarrow B \cap C \subset C \cap B \dots(i)$$

Now suppose x be an element of $C \cap B$

$$\text{Then, } x \in C \cap B$$

$$\Rightarrow x \in C \text{ and } x \in B$$

$$\Rightarrow x \in B \text{ and } x \in C \text{ [by definition of intersection]}$$

$$\Rightarrow x \in B \cap C$$

$$\Rightarrow C \cap B \subset B \cap C \dots(ii)$$

From (i) and (ii) we get

$$B \cap C = C \cap B \text{ [every set is a subset of itself]}$$

Hence proved.

25. Let m be the slope of the required line.

$$\therefore \text{Slope} = m = \tan \theta = \tan\{\tan^{-1}(3)\} = 3$$

$$c = y - \text{intercept} = -4$$

Substituting the values of m and c in the general equation of line $y = mx + c$, we get,

$$y = 3x - 4$$

Hence, the equation of the required line is $y = 3x - 4$.

Section C

26. In a word TRIANGLE, vowels are (A, E, I) and consonants are (G, L, N, R, T).

First, we fix the 5 consonants in alternate positions in 5! ways.

-G-L-N-R-T-

In rest of the six blank positions, three vowels can be arranged in 6P_3 ways.

$$\therefore \text{Total number of different words} = 5! \times {}^6P_3$$

$$= 120 \times \frac{6!}{3!}$$

$$= 120 \times 6 \times 5 \times 4$$

$$= 14400$$

27. Let A(-2, 3, 5), B (1, 2, 3) and C(7, 0, -1) be three given points.

$$\text{Then } AB = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2} = \sqrt{9+1+4} = \sqrt{14}$$



$$BC = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} = \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14}$$

$$AC = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2} = \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$$

Now $AC = AB + BC$

Therefore, A, B, C are collinear.

28. To find: middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is 252.

Formula Used:

$T_{r+1} = {}^nC_r x^{n-r} y^r$ where

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Total number of terms in the expansion is 11

Thus, the middle term of the expansion is T_6 and is given by,

$$T_6 = {}^{10}C_5 \times \left(\frac{2x^2}{3}\right)^5 \times \left(\frac{3}{2x^2}\right)^5$$

$$T_6 = {}^{10}C_5$$

$$T_6 = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2}$$

$$T_6 = 252$$

Thus, the middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$ is 252.

OR

Here, $a = 3x^2$, $b = \frac{-1}{3x}$ and $n = 9$

We have a formula,

$$\begin{aligned} t_{r+1} &= \left(\frac{10}{r}\right) a^{n-r} b^r \\ &= \left(\frac{9}{r}\right) (3x^2)^{9-r} \left(\frac{-1}{3x}\right)^r \\ &= \left(\frac{9}{r}\right) (3)^{9-r} (x^2)^{9-r} \left(\frac{-1}{3}\right)^r (x)^{-r} \\ &= \left(\frac{9}{r}\right) (3x)^{9-r} (x)^{18-2r} \left(\frac{-1}{3}\right)^r (x)^{-r} \\ &= \left(\frac{9}{r}\right) (3x)^{9-r} (x)^{18-2r-r} \left(\frac{-1}{3}\right)^r \\ &= \left(\frac{9}{r}\right) (3x)^{9-r} \left(\frac{-1}{3}\right)^r (x)^{18-3r} \end{aligned}$$

To get coefficient of x^6 we must have,

$$(x)^{18-3r} = x^6$$

$$18 - 3r = 6$$

$$3r = 12$$

$$r = 4$$

$$\text{Therefore, coefficient of } x^6 = \left(\frac{9}{4}\right) (3)^{9-4} (3)^{9-4} \left(\frac{-1}{3}\right)^4$$

$$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \cdot (3)^5 \left(\frac{1}{3}\right)^4$$

$$= 126 \times 3$$

$$= 378$$

Hence the coefficient of $x^6 = 378$.

29. We have to evaluate, $\lim_{x \rightarrow \infty} \left[\frac{x}{\sqrt{4x^2+1}-1} \right]$

Rationalising the denominator:

$$\lim_{x \rightarrow \infty} \left[\frac{x}{(\sqrt{4x^2+1}-1)} \cdot \frac{(\sqrt{4x^2+1}+1)}{(\sqrt{4x^2+1}+1)} \right]$$

$$= \lim_{x \rightarrow \infty} \left[\frac{x(\sqrt{4x^2+1}+1)}{4x^2+1-1} \right]$$

$$= \lim_{x \rightarrow \infty} \left[\frac{\sqrt{4x^2+1}+1}{4x} \right]$$

Dividing the numerator and the denominator by x:

$$\begin{aligned}
& \lim_{x \rightarrow \infty} \left[\frac{\frac{\sqrt{4x^2+1}}{x} + \frac{1}{x}}{4} \right] \\
&= \lim_{x \rightarrow \infty} \left[\frac{\sqrt{\frac{4x^2+1}{x^2} + \frac{1}{x}}}{4} \right] \\
&= \lim_{x \rightarrow \infty} \left[\frac{\sqrt{4 + \frac{1}{x^2} + \frac{1}{x}}}{4} \right] \\
& \quad x \rightarrow \infty \\
&\therefore \frac{1}{x}, \frac{1}{x^2} \rightarrow 0 \\
&= \frac{\sqrt{4}}{4} \\
&= \frac{2}{4} \\
&= \frac{1}{2}
\end{aligned}$$

OR

Given,

$$f(x) = \begin{cases} x^2 + ax + b & , 0 \leq x < 2 \\ 3x + 2 & , 2 \leq x \leq 4 \\ 2ax + 5b & , 4 < x \leq 8 \end{cases}$$

To find $\lim_{x \rightarrow 2} f(x)$

$$\text{L.H.L} = \lim_{x \rightarrow 2-} f(x) = \lim_{x \rightarrow 2-} (x^2 + ax + b)$$

$$= 2^2 + a \cdot 2 + b$$

$$= 2a + b + 4$$

$$\text{R.H.L.} = \lim_{x \rightarrow 2+} f(x) = \lim_{x \rightarrow 2+} (3x + 2)$$

$$= 3 \cdot 2 + 2 = 8$$

Since $\lim_{x \rightarrow 2} f(x)$ exists,

$$\therefore \lim_{x \rightarrow 2-} f(x) = \lim_{x \rightarrow 2+} f(x)$$

$$\Rightarrow 2a + b + 4 = 8$$

$$\Rightarrow 2a + b = 4 \dots (i)$$

To find $\lim_{x \rightarrow 4} f(x)$

$$\text{L.H.L.} = \lim_{x \rightarrow 4-} f(x) = \lim_{x \rightarrow 4-} (3x + 2) = 3 \cdot 4 + 2 = 14$$

and

$$\text{R.H.L} = \lim_{x \rightarrow 4+} f(x) = \lim_{x \rightarrow 4+} (2ax + 5b)$$

$$= 2a \cdot 4 + 5b = 8a + 5b$$

Since $\lim_{x \rightarrow 4} f(x)$ exists.

$$\therefore \lim_{x \rightarrow 4-} f(x) = \lim_{x \rightarrow 4+} f(x)$$

$$\Rightarrow 8a + 5b = 14 \dots (ii)$$

From (i) and (ii)

$$\therefore a = 3, b = -2$$

30. Let the required be $\frac{a}{r}$, a and ar. Product of the G.P. = 125

$$\Rightarrow a^3 = 125$$

$$\Rightarrow a = 5$$

$$\text{Sum of the product in pairs} = 87\frac{1}{2} = \frac{175}{2}$$

We can write based on given conditions,

$$\Rightarrow \frac{a}{r} \times a + a \times ar + ar \times \frac{a}{r} = \frac{175}{2}$$

$$\Rightarrow \frac{a^2}{r} + a^2r + a^2 = \frac{175}{2}$$

Substituting the value of a

$$\Rightarrow \frac{25}{r} + 25r + 25 = \frac{175}{2}$$

$$\Rightarrow 50r^2 + 50r + 50 = 175r$$

$$\Rightarrow 50r^2 + 125r + 50 = 0$$

$$\Rightarrow 25(2r^2 - 5r + 2) = 0$$

$$\Rightarrow 2r^2 - 4r - r + 2 = 0$$

$$\Rightarrow 2r(r - 2) - 1(r - 2) = 0$$

$$\Rightarrow (2r - 1)(r - 2) = 0$$

$$\therefore r = \frac{1}{2}, 2$$

Hence, the G.P. for $a = 5$ and $r = \frac{1}{2}$ is 10, 5 and $\frac{5}{2}$

And, the G.P. for $a = 5$ and $r = 2$ is $\frac{5}{2}$, 5 and 10

OR

Here, it is given the series, $-\frac{3}{4}, \frac{1}{2}, -\frac{1}{3}, \frac{2}{9}, \dots$

The standard form of GP is, a, ar, ar^2, ar^3, \dots

The common ratio is r .

First term in the given GP, $a_1 = a = -\frac{3}{4}$

Second term in GP, $a_2 = \frac{1}{2}$

The common ratio, $r = \frac{a_2}{a_1}, r = -\frac{\frac{1}{2}}{\frac{3}{4}} = -\frac{2}{3}$

Now, n th term of GP is, $a^n = ar^{n-1}$

Therefore, the 10th term, $a^{10} = ar^9$

$$a_{10} = ar^9 = \left(-\frac{3}{4}\right) \left(-\frac{2}{3}\right)^9 = \frac{128}{6561}$$

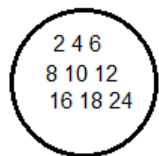
Now, the required n th term, $a^n = ar^{n-1}$

$$a_n = ar^{n-1} = \left(-\frac{3}{4}\right) \left(-\frac{2}{3}\right)^{n-1} = \left(\frac{9}{8}\right) \left(-\frac{2}{3}\right)^n$$

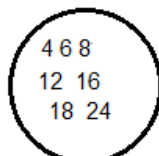
Hence, the 10th term $a_{10} = \frac{128}{6561}$ and the n th term $a_n = \left(\frac{9}{8}\right) \left(-\frac{2}{3}\right)^n$ of the given GP.

$$31. \text{L.H.S: } A \cup B = \{2, 4, 6, 8, 10, 12, 16\},$$

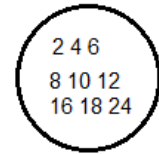
$$(A \cup B) \cup C = \{2, 4, 6, 8, 10, 12, 16, 18, 24\}$$



$$\text{R.H.S: } B \cup C = \{4, 6, 8, 10, 12, 16, 18, 24\}$$



$$A \cup (B \cup C) = \{2, 4, 6, 8, 10, 12, 16, 18, 24\}$$



Hence proved.

Thus L.H.S = R.H.S. [Verified]

Section D

32.	class interval	f	Mid-value, x_i	$u_i = \frac{x_i - 35}{10}$	$f_i u_i$	u_i^2	$f_i u_i^2$
	0-10	11	5	-3	-33	9	99
	10-20	29	15	-2	-58	4	116
	20-30	18	25	-1	-18	1	18

30-40	4	35	0	0	0	0
40-50	5	45	1	5	1	5
50-60	3	55	2	6	4	12
	$N = \sum f_i = 70$			$\sum f_i u_i = -98$		$\sum f_i u_i^2 = 250$

$N = 70$, $\sum f_i u_i = -98$, $\sum f_i u_i^2 = 250$, $A = 35$ and $h = 10$

$$\text{Mean} = A + h \left(\frac{1}{N} \sum f_i u_i \right) = 35 + 10 \left(\frac{-98}{70} \right) = 21$$

$$\text{Var}(X) = h^2 \left\{ \left(\frac{1}{N} \sum f_i u_i^2 \right) - \left(\frac{1}{N} \sum f_i u_i \right)^2 \right\} = 100 \left\{ \left(\frac{1}{70} \times 250 \right) - \left(\frac{1}{70} \times (-98) \right)^2 \right\} = 100 \{ 3.57 - 1.96 \} = 161$$

$$\text{SD} = \sqrt{\text{var}(X)} = \sqrt{161} = 12.69$$

33. Given that:

$$3x^2 + 2y^2 = 18$$

After Divide by 18 to both the sides, we get

$$\frac{3}{18}x^2 + \frac{2}{18}y^2 = 1 \Rightarrow \frac{x^2}{6} + \frac{y^2}{9} = 1 \dots (i)$$

Now, above equation is of the form,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots (ii)$$

Comparing eq. (i) and (ii), we get

$$a^2 = 9 \text{ and } b^2 = 6 \Rightarrow a = \sqrt{9} \text{ and } b = \sqrt{6} \Rightarrow a = 3 \text{ and } b = \sqrt{6}$$

i. Length or major axes

$$\therefore \text{Length of major axes} = 2a = 2 \times 3 = 6 \text{ units}$$

ii. Length or minor axes

$$\therefore \text{Length or minor axes} = 2b = 2 \times \sqrt{6} = 2\sqrt{6}$$

iii. Coordinates of the vertices

$$\therefore \text{Coordinates of vertices} = (0, a) \text{ and } (0, -a) = (0, 6) \text{ and } (0, -6)$$

iv. Coordinates of the foci

$$\text{As we know that, Coordinates of foci} = (0, \pm c)$$

$$\text{where } c^2 = a^2 - b^2$$

Now

$$c^2 = 9 - 6 \Rightarrow c^2 = 3 \Rightarrow c = \sqrt{3} \dots (iii)$$

$$\therefore \text{Coordinates of foci} = (0, \pm\sqrt{3})$$

v. Eccentricity

$$\text{As we know that, Eccentricity} = \frac{c}{a} \Rightarrow e = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \text{ [from (iii)]}$$

vi. Length of the Latus Rectum

$$\begin{aligned} \text{As we know that, Length of Latus Rectum} &= \frac{2b^2}{a} = \frac{2 \times (\sqrt{6})^2}{3} \\ &= \frac{2 \times 6}{3} = 4 \end{aligned}$$

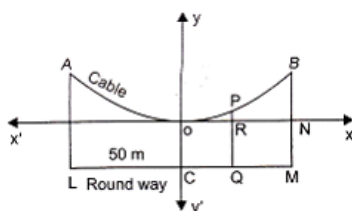
OR

Let AOB be the cable of uniformly loaded suspension bridge. Let AL and BM be the longest wires of length 30 m each. Let OC be the shortest wire of length 6 m and LM be the roadway.

Now $AL = BM = 30$ m, $OC = 6$ m and $LM = 100$ m.

$$\therefore LC = CM = \frac{1}{2} LM = 50 \text{ m}$$

Let O be the vertex and axis of the parabola be y-axis. So the equation of parabola in standard form is $x^2 = 4ay$



Coordinates of point B are (50, 24)

Since point B lies on the parabola $x^2 = 4ay$

$$\therefore (50)^2 = 4a \times 24 \Rightarrow a = \frac{2500}{4 \times 24} = \frac{625}{24}$$

$$\text{So equation of parabola is } x^2 = \frac{4 \times 625}{24} y \Rightarrow x^2 = \frac{625}{6} y$$

Let length of the supporting wire PW at a distance of 18 m be h.

$$\therefore OR = 18 \text{ m and } PR = PQ - QP = PQ - OC = h - 6$$

Coordinates of point P are (18, h - 6)

$$\text{Since the point P lies on parabola } x^2 = \frac{625}{6} y$$

$$\therefore (18)^2 = \frac{625}{6} (h - 6) \Rightarrow 324 \times 6 = 625h - 3750$$

$$\Rightarrow 625h = 1944 + 3750 \Rightarrow h = \frac{5694}{625} = 9.11 \text{ m approx.}$$

$$34. \text{ We have, } \frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4} \dots (i)$$

$$\text{and } \frac{7x-1}{3} - \frac{7x+2}{6} > x \dots (ii)$$

From inequality (i), we get

$$\frac{4x}{3} - \frac{9}{4} < x + \frac{3}{4} \Rightarrow \frac{16x-27}{12} < \frac{4x+3}{4}$$

$$\Rightarrow 16x - 27 < 12x + 9 \text{ [multiplying both sides by 12]}$$

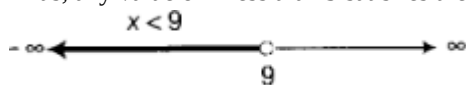
$$\Rightarrow 16x - 27 + 27 < 12x + 9 + 27 \text{ [adding 27 on both sides]}$$

$$\Rightarrow 16x < 12x + 36$$

$$\Rightarrow 16x - 12x < 12x + 36 - 12x \text{ [subtracting 12x from bot sides]}$$

$$\Rightarrow 4x < 36 \Rightarrow x < 9 \text{ [dividing both sides by 4]}$$

Thus, any value of x less than 9 satisfies the inequality. So, the solution of inequality (i) is given by $x \in (-\infty, 9)$



From inequality (ii) we get,

$$\frac{7x-1}{3} - \frac{7x+2}{6} > x \Rightarrow \frac{14x-2-7x-2}{6} > x$$

$$\Rightarrow 7x - 4 > 6x \text{ [multiplying by 6 on both sides]}$$

$$\Rightarrow 7x - 4 + 4 > 6x + 4 \text{ [adding 4 on both sides]}$$

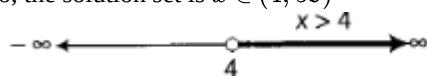
$$\Rightarrow 7x > 6x + 4$$

$$\Rightarrow 7x - 6x > 6x + 4 - 6x \text{ [subtracting 6x from both sides]}$$

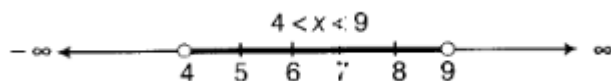
$$\therefore x > 4$$

Thus, any value of x greater than 4 satisfies the inequality.

So, the solution set is $x \in (4, \infty)$



The solution set of inequalities (i) and (ii) are represented graphically on number line as given below:



Clearly, the common value of x lie between 4 and 9.

Hence, the solution of the given system is, $4 < x < 9$ i.e., $x \in (4, 9)$

$$35. \text{ LHS} = \cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15}$$

$$= \cos \frac{2\pi}{15} \cos 2 \left(\frac{2\pi}{15} \right) \cos 4 \left(\frac{2\pi}{15} \right) \cos 8 \left(\frac{2\pi}{15} \right)$$

$$\text{Put } \frac{2\pi}{15} = \alpha$$

$$\Rightarrow \text{LHS} = \cos \alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha$$

$$= \frac{2 \sin \alpha [\cos \alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha]}{2 \sin \alpha} \text{ [multiplying numerator and denominator by } 2 \sin \alpha]$$

$$= \frac{(2 \sin \alpha \cdot \cos \alpha) \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha}{2 \sin \alpha}$$

$$= \frac{2(\sin 2\alpha \cdot \cos 2\alpha \cdot \cos 4\alpha \cdot \cos 8\alpha)}{2(2 \sin \alpha)} \text{ [} \because 2 \sin \alpha \cos \alpha = \sin 2\alpha \text{ and multiplying numerator and denominator by 2]}$$

$$= \frac{(2 \sin 2\alpha \cdot \cos 2\alpha) \cdot \cos 4\alpha \cdot \cos 8\alpha}{4 \sin \alpha}$$

$$= \frac{2(\sin 4\alpha \cdot \cos 4\alpha) \cdot \cos 8\alpha}{2(4 \sin \alpha)} \text{ [} \because 2 \sin \alpha \cos \alpha = \sin 2\alpha \text{ and multiplying numerator and denominator by 2]}$$

$$= \frac{2(\sin 8\alpha \cdot \cos 8\alpha)}{2(8 \sin \alpha)}$$

$$= \frac{\sin 16\alpha}{16 \sin \alpha} = \frac{\sin(15\alpha + \alpha)}{16 \sin \alpha}$$

$$\text{Now, } 15\alpha = 2\pi,$$

$$= \frac{\sin(2\pi + \alpha)}{16 \sin \alpha} = \frac{\sin \alpha}{16 \sin \alpha} = \frac{1}{16} = \text{RHS}$$



∴ LHS = RHS

Hence proved.

OR

We have to find the values of $\cos \frac{x}{2}$, $\sin \frac{x}{2}$ and $\sin 2x$.

It is given that $\cos x = -\frac{3}{5}$ and x lies in the IIIrd quadrant

We know,

$$\cos 2x = 2 \cos^2 x - 1$$

$$\cos x = 2 \cos^2 \frac{x}{2} - 1$$

$$-\frac{3}{5} = 2 \cos^2 \frac{x}{2} - 1 \dots [\because \cos x = -\frac{3}{5}]$$

$$2 \cos^2 \frac{x}{2} = -\frac{3}{5} + 1$$

$$2 \cos^2 \frac{x}{2} = \frac{2}{5}$$

$$\cos^2 \frac{x}{2} = \frac{1}{5}$$

$$\cos \frac{x}{2} = \pm \frac{1}{\sqrt{5}}$$

Since,

$$x \in \left(\pi, \frac{3\pi}{2}\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$$

$\cos \frac{x}{2}$ will be negative in 3rd quadrant

So,

$$\cos x = -\frac{1}{\sqrt{5}}$$

We know,

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\cos x = 1 - 2 \sin^2 \frac{x}{2} \dots [\because \cos x = -\frac{3}{5}]$$

$$-\frac{3}{5} = 1 - 2 \sin^2 \frac{x}{2}$$

$$2 \sin^2 \frac{x}{2} = \frac{3}{5} + 1$$

$$2 \sin^2 \frac{x}{2} = \frac{8}{5}$$

$$\sin^2 \frac{x}{2} = \frac{4}{5}$$

$$\sin \frac{x}{2} = \pm \frac{2}{\sqrt{5}}$$

Since,

$$x \in \left(\pi, \frac{3\pi}{2}\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$$

$\sin \frac{x}{2}$ will be positive in 2nd quadrant

So,

$$\sin \frac{x}{2} = \frac{2}{\sqrt{5}}$$

We know,

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin^2 x = 1 - \left(-\frac{3}{5}\right)^2 \dots [\because \cos x = -\frac{3}{5}]$$

$$\sin^2 x = 1 - \frac{9}{25}$$

$$\sin^2 x = \frac{25-9}{25}$$

$$\sin^2 x = \frac{16}{25}$$

$$\sin x = \pm \frac{4}{5}$$

Since,

$$x \in \left(\pi, \frac{3\pi}{2}\right)$$

$\sin x$ will be negative in 3rd quadrant

So,

$$\sin x = -\frac{4}{5}$$

Now,

$$\sin 2x = 2(\sin x)(\cos x) \dots [\because \cos x = -\frac{3}{5} \text{ and } \sin x = -\frac{4}{5}]$$

$$\sin 2x = 2 \times -\frac{4}{5} \times -\frac{3}{5}$$

$$\sin 2x = \frac{24}{25}$$

Hence, values of $\cos \frac{x}{2}$, $\sin \frac{x}{2}$, $\sin 2x$ are $-\frac{1}{\sqrt{5}}$, $\frac{2}{\sqrt{5}}$ and $\frac{24}{25}$

Section E

36. i. $n(A \times A) = 9$

$$\Rightarrow n(A) \times n(A) = 9 \Rightarrow n(A) = 3$$

$$(-1, 0) \in A \times A \Rightarrow -1 \in A, 0 \in A$$

$$(0, 1) \in A \times A \Rightarrow 0 \in A, 1 \in A$$

$$\Rightarrow -1, 0, 1 \in A$$

$$\text{Also, } n(A) = 3 \Rightarrow A = \{-1, 0, 1\}$$

$$\text{Hence, } A = \{-1, 0, 1\}$$

$$\text{Also, } A \times A = \{-1, 0, 1\} \times \{-1, 0, 1\}$$

$$= \{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)\}$$

Hence, the remaining elements of $A \times A$ are

$$(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0) \text{ and } (1, 1).$$

ii. Given, $(A \times B) = 6$ and $(A \times B) = \{(1, 3), (2, 5), (3, 3)\}$

We know that Cartesian product of set $A = \{a, b\}$ & $B = \{c, d\}$ is $A \times B = \{(a, c), (a, d), (b, c), (b, d)\}$

Therefore, $A = \{1, 2, 3\}$ & $B = \{3, 5\}$

$$\Rightarrow A \times B = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$$

Thus, remaining elements are $A \times B = \{(1, 5), (2, 3), (3, 5)\}$

iii. If the set A has 3 elements and set B has 4 elements, then the number of elements in $A \times B = 12$

OR

Clearly, A is the set of all first entries in ordered pairs in $A \times B$ and B is the set of all second entries in ordered pairs in $A \times B$

$$\therefore A = \{a, b\} \text{ and } B = \{1, 2, 3\}$$

37. i. Let E_1 and E_2 denotes the events that Ankit and Vinod will respectively qualify the exam.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= 0.05 + 0.10 - 0.02 = 0.13$$

ii. Let E_1 and E_2 denotes the events that Ankit and Vinod will respectively qualify the exam.

Probability of atleast one of them does not qualify

$$= P(E_1' \cup E_2') = P((E_1 \cap E_2)')$$

$$= 1 - P(E_1 \cap E_2) = 1 - 0.02 = 0.98$$

iii. Let E_1 and E_2 denotes the events that Ankit and Vinod will respectively qualify the exam.

$$= P(E_1' \cap E_2') = P((E_1 \cup E_2)')$$

$$= 1 - P(E_1 \cup E_2) = 1 - 0.13 = 0.87$$

OR

Let E_1 and E_2 denotes the events that Ankit and Vinod will respectively qualify the exam.

The probability that Vinod will not qualify the exam.

$$\text{Probability that only one of them will qualify the exam} = P((E_1 - E_2) \cup (E_2 - E_1))$$

$$= P(E_1 - E_2) + P(E_2 - E_1)$$

$$= P(E_1 \cup E_2) - P(E_1 \cap E_2)$$

$$= 0.13 - 0.02 = 0.11$$

38. i. Let $z = 1 + 2i$

$$\Rightarrow |z| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\text{Now, } f(z) = \frac{7-z}{1-z^2} = \frac{7-1-2i}{1-(1+2i)^2}$$

$$= \frac{6-2i}{1-1-4i-2-4i} = \frac{6-2i}{-4-4i}$$

$$= \frac{(3-i)(2+2i)}{(2-2i)(2+2i)}$$

$$= \frac{6-2i+6i-2i^2}{4-4i^2} = \frac{6+4i+2}{4+4}$$

$$= \frac{8+4i}{8} = 1 + \frac{1}{2}i$$

$$f(z) = 1 + \frac{1}{z}$$

$$\therefore |f(z)| = \sqrt{1 + \frac{1}{z\bar{z}}} = \sqrt{\frac{4+1}{4}} = \frac{\sqrt{5}}{2} = \frac{|z|}{2}$$

ii. Given that: $(z + 3)(\bar{z} + 3)$

Let $z = x + yi$

$$\text{So } (z + 3)(\bar{z} + 3) = (x + yi + 3)(x - yi + 3)$$

$$= [(x + 3) + yi][(x + 3) - yi]$$

$$= (x + 3)^2 - y^2 i^2$$

$$= (x + 3)^2 + y^2$$

$$= |x + 3 + iy|^2$$

$$= |z + 3|^2$$

iii. The conjugate of $-6 - 24i$ is $-6 + 24i$.

$$\text{It is given that } -6 + 24i = (x - iy)(3 + 5i)$$

$$-6 + 24i = 3x + 5xi - 3iy - 5yi^2$$

$$-6 + 24i = (3x + 5y) + i(5x - 3y)$$

Comparing the real and imaginary parts,

$$3x + 5y = -6$$

$$5x - 3y = 24$$

Solving these two equations we get $x = 3$ and $y = -3$.

Therefore, $x = 3$ and $y = -3$

$$\text{Then } x + y = 3 - 3 = 0$$

OR

$$z = 3 + 4i$$

$$\Rightarrow \bar{z} = 3 - 4i$$